Inverse Kinematic Manipulation Control and Trajectory Tracking of Robotic Arm Using Trigonometric Tangential Mathematical Models

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ABSTRACT
Industrial robot is forced to sustain certain complications of real-world applications that continually influence its motion competency embracing end-effector positional accuracy and repeatability, degree-of-freedom limitation, redundant movement, heavy payload uplifting, long horizontal reach stretching, and others. This work focuses on devising inverse kinematic (IK) solutions to assist the robotic arm motion control using trigonometric tangential mathematical models. The nonlinear mathematical models acquired from the extensive manipulation of trigonometric rules, specifically sum, Pythagorean, and tangent quotient identities and algebraic arithmetic in pursuit of determining the reachable actuating joint configurations are experimented for applicability on the fundamental structure of two-segmented manipulator arm. For verification, the precision of the IK solutions yielded by the models are randomly benchmarked with the manipulator's direct kinematics and tested with the statistical performance measure of minimum squared error while tracking cubic Hermite spline, cubic Bezier, and cubic B-spline curves. For validation, an interactive spreadsheet-based IK application utilizing built-in front-end capabilities including Visual Basic for Applications, Math and Trig Function Library, Name Manager, Data Validation, ActiveX Controls, and Charts is developed to accommodate these models and simulate the feasible joint angles and orientations of robotic arm. The application visualizes (1) the robotic linkage motion on xz plane according to the links lengths, end-effector position, and base position stipulated and (2) the robotic curves trajectory tracking of cubic Hermite spline, cubic Bezier, and cubic B-spline. The trigonometric tangential mathematical models suggested afford a credible and practical IK solution alternative for the manipulation control and trajectory tracking of two-segmented robotic manipulator arm.

Keywords: *Cubic B-spline, Cubic Bezier, Cubic Hermite Spline, Inverse Kinematics, Linkage Motion, Manipulation Control, Robotic Manipulator Arm, Trajectory Tracking, Trigonometric Rule*.

I. INTRODUCTION

Industrial robot composes a serial-linked sophisticated mechanism that imitates the biological anatomy and behavior of human arm for great flexibility, agility, and versatility while performing tasks [1]. As an indispensable technology in manufacturing automation, the manipulator arm is vastly acknowledged for its superior qualities of speed, agility, accuracy, repeatability, and reliability [2]. The technology is popular as it can be reprogrammed and retooled to execute multitasks at relatively low cost [3]. Jobs in hazardous environment that no human can intervene, tedious manual jobs that are vulnerably exposed to human errors, and jobs of repetitive functions that human operators easily get bored with are competently handled by this automated control technology. The robotic manipulator arm has ever become the significant automation enabler to a wide range of processing such as welding, stamping, mounting, spray painting, casting, cutting, grinding, and deburring; assembly such as palletizing, pick-and-place, packaging, loading-unloading, and micro-assembly; and inspections [4]. Safety-critical jobs in which corrective repairs, preventive maintenance, and unexpected reconfigurations are not possible [5] such as nuclear plant installation, nuclear waste remediation, and underwater [6] and aerospace [7] explorations also appreciate overwhelming benefits as a result of the technological elegance of robotic arm variants.

One of the key interests of manipulator arm among robotic experts is related to the science of geometric motion control coined as kinematics, especially inverse kinematics (IK). This is due to the vital foundation of IK in analyzing the design and control of manipulator arm [8, 9, 10]. For instance, instructing the manipulator arm to abide by the target position defined demands its motion to be controlled with the highest stability and accuracy [11]. For the majority of tasks assigned, the manipulator arm usually requires position and orientation data either via leadthrough programming applying teach pendant or simulation using computer-aided design and manufacturing systems [12]. For micro-level processing and assembly, end-effector positional sensitivity is extremely crucial [13]. There are circumstances in which the manipulator arm is compelled to elevate heavy payloads or stretch long horizontal reach within its feasible joint angle limits [14]. Real-life robotic arm applications frequently have to deal with higher degrees-of-freedom (DOF) and redundant motion aside from the naturally serial link-joint chain inaccuracy [15]. All these scenarios claim for the solution of IK, specifically the determination of joint displacements or joint angles effectively and efficiently at any point of time by the robotic manipulator arm controller.

Regardless of all the persuasive significance and usefulness, the IK of manipulator arm oppositely constitutes a complex optimization problem [16] and is deemed as one of the most formidable problems in robotics [14] as it directly dictates accurate motion control for desired trajectory [17, 18]. The complicated IK characteristics as a multi-input multi-output system are substantially dependent on transparent and complete mathematical descriptions for precise representations [19]. The IK complexity originates in the volume of joint variables [20], the diversity of robot configurations [21], the high-nonlinear analytical equations created from the transition of task space to the joint space [22, 23], and the geometric placement of joint axes [24]. The difficulty is also rooted from locating the global optima of multiple optimal local solutions caused by the natural uniqueness of IK problem [25].

Jacobian matrix was postulated to search for the IK solution of 7DOF Mitsubishi PA-10-7C redundant manipulator and the matrix was observed to own the property of high precision and fast convergence [26]. The analytical approach of trigonometric function derivatives and Cramer's rule computed the joint angles of 2R robot respectively on quartic polynomial [27] and cubic Hermite spline curve [15] trajectories. Support vector regression model was more accurately presenting the behavior of 7DOF K-1207 robotic arm than polynomial model as it could afford to cover the larger partitions of arm workspace [28]. New Inverse Kinematics Algorithm of numerical method was introduced to overcome the IK problem of offset-wrist 6DOF robot arms not having nonlinear closed-analytical equations [14]. The FNNs inspired by cerebellar cortex anatomy and function were applied as the IK solution model of two-segmented arm [18]. Contrary to standard genetic algorithm (GA), continuous GA displayed more rapid convergence to the IK solutions of 3R planar manipulator arm in tandem with the minimum execution time and number of generations resulted and the smooth solution curve of joint angles path that minimized the joints net displacement [29]. Generalized reduced gradient algorithm was applied to build a feed-forward neural network well-fitting to predict accurately the joint angles of 2DOF planar manipulator arm [25]. The unified analytical models driven by trigonometric rules and linear system were able to materialize desirable joint angle configurations for manipulating two-segmented arm [24]. The objective of this work is to treat the IK and trajectory problems of robotic manipulator arm with the mathematical models of relatively simple in formulation yet adequately accurate in computation. The work contributes a distinctive theoretical impact as explicit mathematical models are generally more advantageous than implicit models with respect to common modeling issues of underfitting, overfitting, and local optima entrapment.

II. METHOD

This study lays emphasis on trigonometric tangential mathematical models to compute the feasible actuator joint angles and end-effector orientations, Θ of two-segmented planar manipulator arm, simulate its potential linkage motions, and track several spline curve trajectories. The development modus operandi commences by solving the second joint angles, θ_2 prior to the first joint angles, θ_1. Both divisions of the nonlinear analytical models proposed are originated from the product of transformation matrices, ${}^{i-1}T_i$ (1):

$$ {}^{i-1}T_i = \begin{bmatrix} c_i & -\gamma_i s_i & \sigma_i s_i & l_i c_i \\ s_i & \gamma_i c_i & -\sigma_i c_i & l_i s_i \\ 0 & \sigma_i & \gamma_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_i & \mathbf{p}_i \\ 000 & 1 \end{bmatrix} \quad (1) $$

in which $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $\gamma_i = \cos\alpha_i$, $\sigma_i = \sin\alpha_i$, d_i is link offset, l_i is link length, α_i is link twist, R_i is rotational matrix, and $\mathbf{p}_i$ is positional vector.

${}^{0}T_1 \cdot {}^{1}T_2 = P_2$ in which P is end-effector pose matrix (2).

$$ \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$= \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1c_1+l_2c_{12} \\ s_{12} & c_{12} & 0 & l_1s_1+l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & b_x & t_x & p_x \\ n_z & b_z & t_z & p_z \\ n_y & b_y & t_y & p_y \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{n} & \mathbf{b} & \mathbf{t} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

in which $p_x = x_{end}$ is end-effector abscissa and $p_z = z_{end}$ is end-effector applicate.

A. Development of Trigonometric Tangential Mathematical Models

The former section of trigonometric tangential mathematical model is meant for generating the prospective solutions of second joint angle, θ_2. This nonlinear mathematical model (6) is acquired from the manipulation of trigonometric rules, particularly sum, Pythagorean, and tangent quotient identities joining squaring and addition arithmetic.

Stemming from the end-effector positional vector of pose matrix, the trigonometric sum identities, $s_{\alpha\beta} = s_\alpha c_\beta + c_\alpha s_\beta$ and $c_{\alpha\beta} = c_\alpha c_\beta - s_\alpha s_\beta$ are applied; the equations drawn are squared and added; and the trigonometric Pythagorean identity, $s_\theta^2 + c_\theta^2 = 1$ is adopted:

$$x_{end} = l_1c_1 + l_2c_{12} = l_1c_1 + l_2(c_1c_2 - s_1s_2) = l_1c_1 + l_2c_1c_2 - l_2s_1s_2$$

$$z_{end} = l_1s_1 + l_2s_{12} = l_1s_1 + l_2(s_1c_2 + c_1s_2) = l_1s_1 + l_2s_1c_2 + l_2c_1s_2$$

$$x_{end}^2 = (l_1c_1 + l_2c_1c_2 - l_2s_1s_2)^2 = l_1^2c_1^2 + l_2^2c_1^2c_2^2 + l_2^2s_1^2s_2^2 + 2l_1l_2c_1^2c_2 - 2l_1l_2c_1s_1s_2 - 2l_2^2c_1s_1c_2s_2$$

$$z_{end}^2 = (a_1s_1 + l_2s_1c_2 + l_2c_1s_2)^2 = l_1^2s_1^2 + l_2^2s_1^2c_2^2 + l_2^2c_1^2s_2^2 + 2l_1l_2s_1^2c_2 + 2l_1l_2c_1s_1s_2 + 2l_2^2c_1s_1c_2s_2$$

$$x_{end}^2 + z_{end}^2 = (l_1^2c_1^2 + l_2^2c_1^2c_2^2 + l_2^2s_1^2s_2^2 + 2l_1l_2c_1^2c_2 - 2l_1l_2c_1s_1s_2 - 2l_2^2c_1s_1c_2s_2)$$

$$+(l_1^2s_1^2 + l_2^2s_1^2c_2^2 + l_2^2c_1^2s_2^2 + 2l_1l_2s_1^2c_2 + 2l_1l_2c_1s_1s_2 + 2l_2^2c_1s_1c_2s_2)$$

$$= l_1^2c_1^2 + l_1^2s_1^2 + l_2^2c_1^2c_2^2 + l_2^2s_1^2c_2^2 + l_2^2s_1^2s_2^2 + l_2^2c_1^2s_2^2 + 2l_1l_2c_1^2c_2 + 2l_1l_2s_1^2c_2$$

$$= l_1^2(s_1^2 + c_1^2) + l_2^2(s_1^2 + c_1^2)(s_2^2 + c_2^2) + 2l_1l_2c_2(s_1^2 + c_1^2) = l_1^2 + l_2^2 + 2l_1l_2c_2$$

(3)

$$c_2 = \frac{x_{end}^2 + z_{end}^2 - l_1^2 - l_2^2}{2l_1l_2} = \frac{(x_{end} - x_{home})^2 + (z_{end} - z_{home})^2 - l_1^2 - l_2^2}{2l_1l_2}$$

(4)

Equation (4) is streamlined by accounting for the robot base position is not at the origin, $\mathbf{0}$; thus, displacement exists.

The Pythagorean identity $s_\theta^2 + c_\theta^2 = 1$ is employed again to trace any remaining candidates of second joint angle, θ_2:

$$s_2^2 = 1 - c_2^2$$

$$s_2 = \pm\sqrt{1 - c_2^2} \qquad (5)$$

For the straightforward seeking of all second joint angles, θ_2 with only one single model (6), the tangent quotient identity is finally utilized to consolidate equations (4) and (5).

$$t_2 = \frac{s_2}{c_2} = \frac{\pm 2l_1l_2\sqrt{1 - c_2^2}}{(x_{end} - x_{home})^2 + (z_{end} - z_{home})^2 - l_1^2 - l_2^2}$$

$$\theta_2 = \begin{bmatrix} \theta_2^{(1)} & \theta_2^{(2)} & \theta_2^{(3)} & \theta_2^{(4)} \end{bmatrix}^T = t_2^{-1}\left[\frac{\pm 2l_1l_2\sqrt{1 - c_2^2}}{(x_{end} - x_{home})^2 + (z_{end} - z_{home})^2 - l_1^2 - l_2^2}\right]$$

(6)

The latter part of nonlinear models is devised to treasure the possible solutions of first joint angle, θ_1. Rooting from the end-effector positional vector and holding the values of second joint angle, θ_2, the model (13) is established from the usage of trigonometric sum identities, $s_{\alpha\beta} = s_\alpha c_\beta + c_\alpha s_\beta$ and $c_{\alpha\beta} = c_\alpha c_\beta - s_\alpha s_\beta$ and the substitution of equation (9) derived from equation (7), Pythagorean identity, $s_\theta^2 + c_\theta^2 = 1$, and equation (3) into equation (8):

$$x_{end} = l_1c_1 + l_2c_{12} = l_1c_1 + l_2(c_1c_2 - s_1s_2) = l_1c_1 + l_2c_1c_2 - l_2s_1s_2 \qquad (7)$$

$$z_{end} = l_1s_1 + l_2s_{12} = l_1s_1 + l_2(s_1c_2 + c_1s_2) = l_1s_1 + l_2s_1c_2 + l_2c_1s_2 \qquad (8)$$

$$s_1 = \frac{l_1c_1 + l_2c_1c_2 - x_{end}}{l_2s_2} \qquad (9)$$

$$z_{end} = l_1\left(\frac{l_1c_1 + l_2c_1c_2 - x_{end}}{l_2s_2}\right) + l_2\left(\frac{l_1c_1 + l_2c_1c_2 - x_{end}}{l_2s_2}\right)c_2 + l_2c_1s_2$$

$$z_{end}l_2s_2 = (l_1^2c_1 + l_1l_2c_1c_2 - x_{end}l_1) + (l_1l_2c_1c_2 + l_2^2c_1c_2^2 - x_{end}l_2c_2) + (l_2^2c_1s_2^2)$$

$$= c_1(l_1^2 + l_1l_2c_2 + l_1l_2c_2 + l_2^2c_2^2 + l_2^2s_2^2) - x_{end}(l_1 + l_2c_2)$$

$$= c_1[l_1^2 + l_1l_2c_2 + l_1l_2c_2 + l_2^2c_2^2 + l_2^2(1 - c_2^2)] - x_{end}(l_1 + l_2c_2)$$

$$= c_1(l_1^2 + l_1 l_2 c_2 + l_1 l_2 c_2 + l_2^2 c_2^2 + l_2^2 - l_2^2 c_2^2) - x_{end}(l_1 + l_2 c_2)$$

$$= c_1(l_1^2 + l_2^2 + 2 l_1 l_2 c_2) - x_{end}(l_1 + l_2 c_2)$$
$$= c_1(x_{end}^2 + z_{end}^2) - x_{end}(l_1 + l_2 c_2)$$
$$c_1 = \frac{x_{end}(l_1 + l_2 c_2) + z_{end} l_2 s_2}{x_{end}^2 + z_{end}^2} = \frac{(x_{end} - x_{home})(l_1 + l_2 c_2) + (z_{end} - z_{home}) l_2 s_2}{(x_{end} - x_{home})^2 + (z_{end} - z_{home})^2} \quad (10)$$

Equation (10) is reordered with the consideration that displacement may exist should the robot base position is not located at the origin, **O**.

The replacement of equation (11) exploited from equation (8), Pythagorean identity, $s_\theta^2 + c_\theta^2 = 1$, and equation (3) into equation (7) is correspondingly used to search further the residual candidates of first joint angle, θ_1:

$$c_1 = \frac{z_{end} - l_1 s_1 - l_2 s_1 c_2}{l_2 s_2} \quad (11)$$

$$x_{end} = l_1\left(\frac{z_{end} - l_1 s_1 - l_2 s_1 c_2}{l_2 s_2}\right) + l_2\left(\frac{z_{end} - l_1 s_1 - l_2 s_1 c_2}{l_2 s_2}\right) c_2 - l_2 s_1 s_2$$

$$x_{end} l_2 s_2 = (z_{end} l_1 - l_1^2 s_1 - l_1 l_2 s_1 c_2) + (z_{end} l_2 c_2 - l_1 l_2 s_1 c_2 - l_2^2 s_1 c_2^2) - l_2^2 s_1 s_2^2$$

$$= s_1(-l_1^2 - l_1 l_2 c_2 - l_1 l_2 c_2 - l_2^2 c_2^2 - l_2^2 s_2^2) + z_{end}(l_1 + l_2 c_2)$$

$$= s_1[-l_1^2 - l_1 l_2 c_2 - l_1 l_2 c_2 - l_2^2 c_2^2 - l_2^2(1 - c_2^2)] + z_{end}(l_1 + l_2 c_2)$$

$$= s_1(-l_1^2 - l_1 l_2 c_2 - l_1 l_2 c_2 - l_2^2 c_2^2 - l_2^2 + l_2^2 c_2^2) + z_{end}(l_1 + l_2 c_2)$$

$$= s_1(-l_1^2 - l_2^2 - 2 l_1 l_2 c_2) + z_{end}(l_1 + l_2 c_2)$$
$$= -s_1(l_1^2 + l_2^2 + 2 l_1 l_2 c_2) + z_{end}(l_1 + l_2 c_2)$$
$$= -s_1(x_{end}^2 + z_{end}^2) + z_{end}(l_1 + l_2 c_2)$$
$$s_1 = \frac{z_{end}(l_1 + l_2 c_2) - x_{end} l_2 s_2}{x_{end}^2 + z_{end}^2} = \frac{(z_{end} - z_{home})(l_1 + l_2 c_2) - (x_{end} - x_{home}) l_2 s_2}{(x_{end} - x_{home})^2 + (z_{end} - z_{home})^2} \quad (12)$$

Since the robot base position is not always at the origin, **O**; adjustment to equation (12) is essential to cater for such displacement.

Finding the first joint angles, θ_1 also simply requires one single model (13) after lastly integrating derivations (10) and (12) via the tangent quotient identity:

$$t_1 = \frac{s_1}{c_1} = \frac{(z_{end} - z_{home})(l_1 + l_2 c_2) - (x_{end} - x_{home}) l_2 s_2}{(x_{end} - x_{home})(l_1 + l_2 c_2) + (z_{end} - z_{home}) l_2 s_2}$$

$$\theta_1 = \left[\theta_1^{(1)} \quad \theta_1^{(2)} \quad \theta_1^{(3)} \quad \theta_1^{(4)}\right]^T = t_1^{-1}\left[\frac{(z_{end} - z_{home})(l_1 + l_2 c_2) - (x_{end} - x_{home}) l_2 s_2}{(x_{end} - x_{home})(l_1 + l_2 c_2) + (z_{end} - z_{home}) l_2 s_2}\right] \quad (13)$$

Figure 1 summarizes the alternative trigonometric tangential mathematical models undertaken for computing the IK solutions of two-segmented robotic manipulator arm:

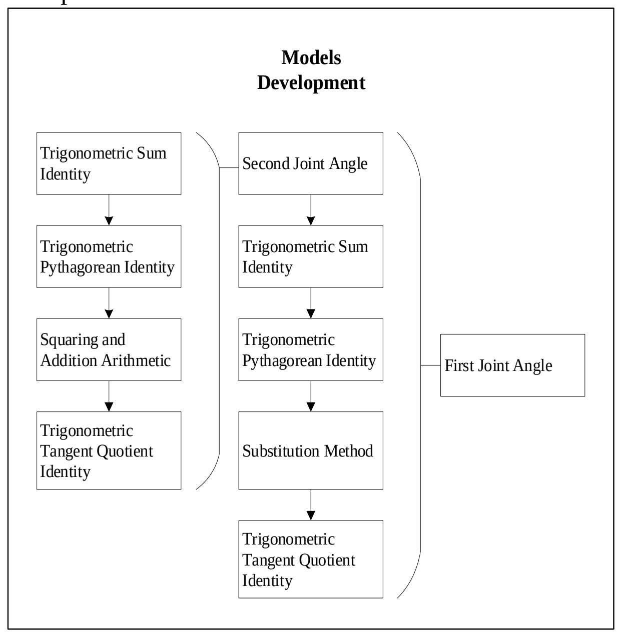


Figure 1: Development of trigonometric tangential mathematical inverse kinematic models

III. RESULTS AND DISCUSSION

A. Verification with Direct Kinematic Model

For the intent of verification, the precision of inverse kinematic (IK) solutions by the trigonometric tangent mathematical models is arbitrarily cross-referenced with the direct kinematics (DK) of manipulator arm. It is a good practice to double-check the IK solutions procured to the DK formulation for verification [20]. All solutions of IK must be examined to verify whether they may feasibly drive the end-effector to the desired position [8]. Table 1 presents the results obtained from the IK models developed and their comparison with the DK:

Table 1: Matching of IK models’ results to DK

DK					**IK**	
					(6)	**(13)**
l_1 l_2	θ_1	θ_2	x_{home} z_{home}	x_{end} z_{end}	θ_2	θ_1
1 3.0 5. 0	7. 0 2 7.0	37. 0 32 3.0	0.0 0 0.0 0	16. 50 5.0 6	*37.0* 217. 0 143. 0 *323.* *0*	*7.0* 187.0 *27.1* 207.1
2	1	21.	1.0	30.	*21.0*	*14.0*

4.0 7. 0	4.0 2 3.0	0 33 9.0	0 2.0 0	02 11. 82	201. 0 159. 0 *339.* *0*	194.0 *23.4* 203.4
1 5.0 8. 0	5 3.0 6 9.0	23. 0 33 7.0	2.0 0 3.0 0	12. 96 22. 74	*23.1* 203. 1 156. 9 *336.* *9*	*53.0* 233.0 *68.9* 248.9
2 6.0 1 0.0	3 4.0 6 3.0	54. 0 30 6.0	3.0 0 3.0 0	24. 90 27. 53	*54.0* 234. 0 126. 0 *306.* *0*	*34.0* 214.0 *62.5* 242.5
4 8.0 1 4.0	5 4.0 5 7.0	7.0 35 3.0	4.0 0 5.0 0	39. 00 56. 08	*6.9* 186. 9 173. 1 *353.* *1*	*54.0* 234.0 *57.1* 237.1

The italic results in Table 1 proves the efficacy of trigonometric tangential mathematical models (6) and (13) in resolving the IK problem of two-segmented planar manipulator arm. Although minor deviations of joint angle vectors, $[\theta_1 \theta_2]^T$ occur, the results computed by the models are acceptable due to minimum errors.

B. Verifications with Cubic Hermite Spline, Cubic Bezier, and Cubic B-Spline Curve Models

Aside from the direct kinematic fitting, the reliability of proposed models is experimented against the curve trajectory tracking of cubic Hermite spline (14), cubic Bezier (15), and cubic B-spline (16). The models are found high-performing based on the very low mean squared errors (MSE) (17) illustrated in Table 2 respectively through below and above configurations:

$$P_{curve}(u) = (x_{curve}, z_{curve}) = P_{ctrl1}(2u^3 - 3u^2 + 1) + P_{ctrl2}(u^3 - 2u^2 + u) + P_{ctrl3}(u^3 - u^2) + P_{ctrl4}(-2u^3 + 3u^2) \quad (14)$$

$$P_{curve}(u) = (x_{curve}, z_{curve}) = P_{ctrl1}(-u^3 - 3u^2 + 1) + P_{ctrl2}(3u^3 - 6u^2 + 3u) + P_{ctrl3}(-3u^3 + 3u^2) + P_{ctrl4}(u^3) \quad (15)$$

$$P_{curve}(u) = (x_{curve}, z_{curve}) = P_{ctrl1}\left(\frac{-u^3+3u^2-3u+1}{6}\right) + P_{ctrl2}\left(\frac{3u^3-6u^2+4}{6}\right) + P_{ctrl3}\left(\frac{-3u^3-6u^2+4}{6}\right) + P_{ctrl4}\left(\frac{u^3}{6}\right) \quad (16)$$

in which P_{ctrl} is control point and $u = [0,1]$.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left[\left(x_{curve}^{(i)} - x_{end}^{(i)}\right)^2 + \left(z_{curve}^{(i)} - z_{end}^{(i)}\right)^2\right] \quad (17)$$

Table 2: Curve trajectory tracking results

Type of Curve	Joint Configuration	MSE
Cubic Hermite Spline	Elbow-down	4×10^{-4}
	Elbow-up	5×10^{-4}
Cubic Bezier	Elbow-down	5×10^{-4}
	Elbow-up	6×10^{-4}
Cubic B-spline	Elbow-down	6×10^{-4}
	Elbow-up	6×10^{-4}

Figure 2 exhibits the verification processes involved in the trigonometric tangential mathematical models.

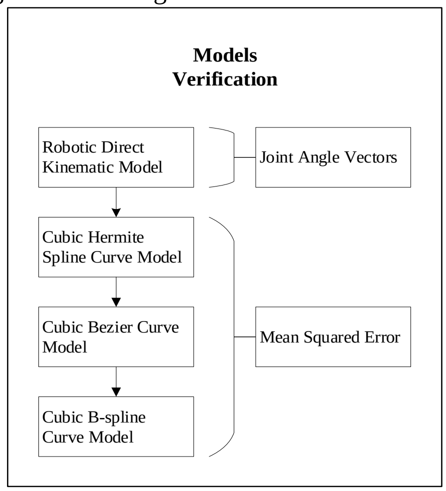
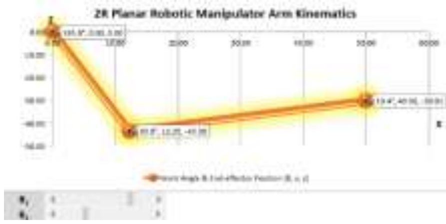


Figure 2: Verification of trigonometric tangential mathematical inverse kinematic models

C. Validation with Spreadsheet Computing and Simulation

A spreadsheet-based inverse kinematic (IK) application is constructed for the computation and simulation of trigonometric tangential mathematical models. Simulation is an effective virtual experiment to test diversified models or algorithms for manipulator arm [26]. The application makes use of inbuilt front-end features such as Visual Basic for Applications for the models programming, Math and Trig Function Library, Name Manager for variables definition and management, Data Validation for incorrect inputs prevention, ActiveX Controls for user-friendly data input, and Charts for data analysis description. This application is able to justify the validation purpose, expressly to compute all candidates of joint angles, simulate the feasible ones with orientation, and visualize the linkage motion of manipulator arm on xz plane in accordance with the links lengths, end-effector position, and base position specified.

Figures 3(a) and 3(b) visualize the simulated linkage motions of two-segmented manipulator arm:



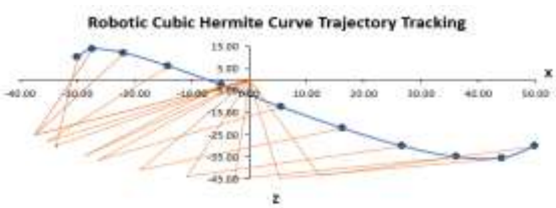
(a)

(b)

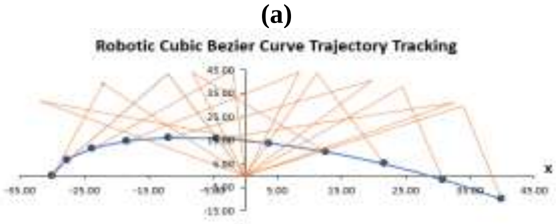
Figure 3: Linkage motion simulation visual of two-segmented manipulator arm

From the validation process, it is learned that two settings of linkage motion are accomplishable for the two-segmented manipulator arm. Figures 3(a) and 3(b) are individually recognized as the elbow-down and elbow-up configurations.

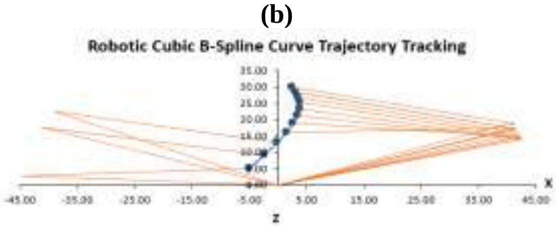
On top of the linkage motion, the validation of trigonometric tangential mathematical models is also conducted through the simulated curve trajectory tracking of cubic Hermite spline, cubic Bezier, and cubic B-spline as depicted in Figure 4:



(a)



(b)



(c)

Figure 4: Trajectory tracking simulation visual of two-segmented manipulator arm

Figures 4(a), 4(b), and 4(c) evidently indicate the models are capable to maneuver the manipulator arm to contact the cubic Hermite spline, cubic Bezier, and cubic B-spline curve points precisely in both the elbow-down and elbow-up formations.

The validation activities concerned in the trigonometric tangential mathematical models are shown in Figure 5.

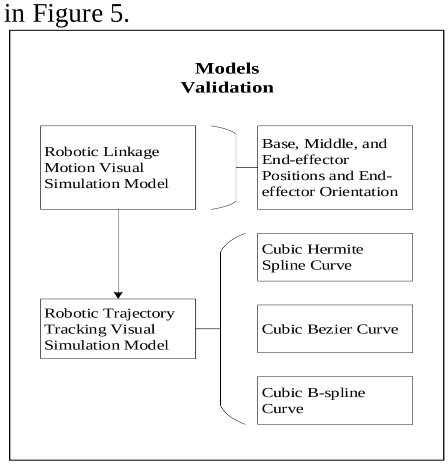


Figure 5: Validation of trigonometric tangential mathematical inverse kinematic models

IV. CONCLUSION

The sum, Pythagorean, and tangent quotient identities of trigonometric rule are extensively harnessed to realize the trigonometric tangent mathematical models. The inverse kinematic (IK) solutions produced by these models are discovered feasible and precise after randomly verifying with the direct kinematic models and validating with the spreadsheet computing and simulation for the robotic linkage motion and curve trajectory tracking. The nonlinear analytical models are also learned robust and practical in formulation and computation as the nonlinearity is merely the simplified quadratic form.

Extended studies to enrich the values of this work cover the development and deployment of new analytical models using transformation matrix or Jacobian matrix inversion for the IK solutions of spatial robotic manipulator arm. Planning to attempt for varied solution paradigms involving geometrical principles, statistical procedures, numerical methods, and soft computing techniques are also accounted for to improve the near future works.

APPENDIX

Appendixes, if needed, appear before the acknowledgment.

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